REPORT 352

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SOME STATIC AEROELASTIC CONSIDERATIONS OF SLENDER AIRCRAFT

by

G. J. HANCOCK

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(6) SOME STATIC AEROELASTIC CONSIDERATIONS OF SLENDER AIRCRAFT,

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SUMMARY

A series of simple theoretical models are discussed, representing the essential features of flexible aircraft both of the classical and integrated varieties, in trimmed level flight. Assuming linear aerodynamics it is shown that the maximum speed for trimmed flight for the classical aircraft is determined primarily by the tail-plane flexibility. For the integrated aircraft this maximum speed occurs when both the overall aeroelastic distortion and the control forces become very large, the effectiveness of the control to trim is related to the similarity of the weight and aerodynamic lifting distributions. Detailed calculations on trimmed plate wings of varying plan form further illustrate these general points.

The aeroelastic effects of these trimmed plate wings in level flight are also investigated assuming non-linear aerodynamics. It is shown that in general there are two positions of equilibrium of the aircraft at each speed although it is possible that at high speeds both these positions of equilibrium are imaginary. The stability of these positions of equilibrium are not discussed in this paper.

SOMMAIRE

Ce rapport traite d'une série de modèles théoriques simples représentant les caractéristiques fondamentales des avions souples, tant classiques qu'intégrés, dans le vol équilibré en palier. En supposant l'aérodynamique linéaire, il est démontré que la vitesse maximum du vol équilibré de l'avion classique est déterminée principalement par la souplesse de l'empennage. En ce qui concerne l'appareil intégré, cette vitesse maximum se produit lorsque la distorsion aéroélastique globale ainsi que les forces des gouvernes deviennent très importantes: l'efficacité des gouvernes en vue de l'équilibrage est fonction de la similitude des répartitions du poids et de la aérodynamique. Des calculs détaillés relatifs à des ailes équilibrées de formes en plan diverses mettent plus en évidence ces points généraux.

Les effets aéroélastiques de ces ailes équilibrées pour le vol en palier sont également étudiés en supposant l'aérodynamique nonlinéaire. Il est démontré que de façon générale il existe deux positions d'équilibre pour l'avion à chaque vitesse, bien qu'il soit possible qu'à des vitesses élevées ces deux positions d'équilibre soient imaginaires. La stabilité de ces positions d'équilibre n'est pas discutée dans cette communication.

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SOME STATIC AEROELASTIC CONSIDERATIONS OF SLENDER AIRCRAFT

G.J. Hancock*

1. INTRODUCTION

The techniques for the calculation of aeroelastic effects have been formulated and perfected during the evolution of the conventional, or classical aircraft, comprised of distinct wings, fuselage and tail and acted on by aerodynamic forces which depend linearly on incidence. We are now in a state of transition from this conventional subsonic aircraft to the supersonic aircraft of the future. The latter is expected to differ from the former in two important respects. First, the supersonic configuration will be integrated into a single unit combining the roles of wings, fuselage and tail. Second, the current supersonic aerodynamic design philosophy of incorporating flow separations and the formation of leading-edge vortices as essential parts of the flow field implies that the aerodynamic forces will be non linear functions of incidence. Therefore, from an aeroelastic point of view two important questions may be posed. First, which of our aeroelastic concepts based on background of knowledge built up with the experience on conventional aircraft, may be safely extrapolated to the integrated aircraft, and which of our concepts have to be thought about again and re-defined? Secondly, what are the additional aeroelastic effects due to the presence of the non linear aerodynamic forces? In this Report we only begin to answer these questions by consideration of some simple static aeroelastic effects.

As an indication of the reality and depth of these questions, a difficulty arises straight away over definitions. In the past, static aeroelasticity has been defined in terms of structural and aerodynamic forces only. This definition tends to lose its meaning when applied to the integrated configuration, so the following definitions are suggested. Static aeroelasticity is the domain of aeroelasticity in which all the forces (inertial, structural and aerodynamic) are present but are independent of time. Dynamic aeroelasticity is the domain of aeroelasticity in which all the forces (inertial, structural and aerodynamic) are present but are dependent on time. So static aeroelasticity refers to the equilibrium problem of a steady manoeuvre, dynamic aeroelasticity refers to stability and response problems. Static stability is a limiting case of dynamic stability and is not included under the present definition of static aeroelasticity.

In this Report we consider only static aeroelasticity, that is, the equilibrium problem, involving a steady controlled manoeuvre. Since one manoeuvre is as representative as any other we shall limit our discussion to the trimmed level flight condition. Since we are interested in relating the aeroelastic effects on conventional aircraft to those on the slender integrated aircraft we shall first review the basic aeroelastic effects on conventional aircraft by reference to some extremely simple models which idealise the various flexibilities of this type of aircraft. Then we shall look at some overall effects of the integrated configurations, again by reference

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to simple models and in this case we shall include a consideration of the non linear aerodynamics.

2. MODEL 1. REPRESENTING THE CLASSICAL AIRCRAFT WITH A FLEXIBLE WING

An attempt is made in this model to illustrate some of the main ideas regarding wing distortion on a conventional aircraft consisting of separate wings, fuselage and tail unit. It is assumed that the wings, fuselage and tail are all rigid, but that the mounting of the main wings to the fuselage is elastic, allowing the wings to have the single degree of freedom of twist only. The torsional stiffness of the wing (of area S_w) relative to the fuselage is denoted by m_θ (lb ft/rad) . The incidence of the fuselage is denoted by α . The angular distortion of the wing relative to the fuselage is denoted by θ , assuming that $\theta = 0$ corresponds to the unloaded equilibrium attitude. This configuration is shown in Figure 1.

For the purposes of this model it is assumed that C_{m_0} is zero. This assumption will not invalidate the qualitative conclusions derived from this model. Therefore in steady trimmed level flight

$$\text{Wing Lift} = L_w = W\left(\frac{l_T}{l_w + l_T}\right) = \tfrac{1}{2}\rho V^2 S_w a_0 (\alpha + \theta) \qquad (1)$$

$$\text{Tail Lift} = L_T = W\left(\frac{l_w}{l_w + l_T}\right) \qquad (2)$$

$$\text{Torsional Distortion} = \theta = \frac{L_w ec}{m_\theta} \qquad (3)$$

$$\text{Fuselage Incidence} = \alpha = \frac{L_w}{\tfrac{1}{2}\rho V^2 S_w a_0}\left[1 - \frac{\tfrac{1}{2}\rho V^2 S_w ec\, a_0}{m_\theta}\right] \qquad (4)$$

(The notation is given in Fig.1). From the expression for the tail lift L_T the elevator angle to trim can be calculated. Thus in this model the angular distortion θ is independent of speed but the fuselage incidence α decreases with speed.

Thus from a *trim point of view* wing distortion is not a critical parameter. This is true even when more sophisticated models, representing wing flexibilities in a more practical manner, are assumed.

3. MODEL 2. REPRESENTING THE CLASSICAL AIRCRAFT WITH A FLEXIBLE FUSELAGE

In this model we look at the flexibility of the fuselage. Again to illustrate the main points we consider an idealized version of the problem. The fuselage is

represented by an elastic beam of uniform cross section and uniform weight distribution (σ lb/ft) . The lifts on the rigid wing and tail (L_W and L_T) together with the wing and tail weights (W_W and W_t) act at the ends of the fuselage. We are neglecting the part of the fuselage ahead of the wings and aft of the tail. The wing section is assumed to be symmetrical (i.e. angle of zero lift is zero) with its centre line parallel to the local fuselage centre line, i.e. the wing incidence is the same as the incidence of the front of the fuselage. This is shown in Figure 2.

The steady trimmed level flight condition gives

$$L_W = W_W + \tfrac{1}{2}\sigma l = \tfrac{1}{2}\rho V^2 S_W a_0 \alpha_{x=0} \tag{5}$$

$$L_T = W_T + \tfrac{1}{2}\sigma l = \tfrac{1}{2}\rho V^2 S_T \left[a_1\left(\alpha_{x=l} - \frac{\partial\epsilon}{\partial\alpha}\alpha_{x=0}\right) + a_2\eta\right] \tag{6}$$

The fuselage distortion $f(x)$ may be found by solving the differential equation for the loaded beam with the appropriate end conditions. The main results however may be deduced directly from physical conditions. These are (i) since L_W is constant then $(\alpha)_{x=0}$ will be the same whether the beam fuselage is rigid or flexible, it is independent of the beam flexibility; (ii) since the loading in the trimmed state is independent of speed the fuselage distortion, measured relative to the axis $(\alpha)_{x=0}$, is independent of speed. Thus from a *trim point of view* fuselage flexibility is not a critical parameter.

4. MODEL 3. REPRESENTING THE CLASSICAL AIRCRAFT WITH A FLEXIBLE TAIL

To complete our analysis of the conventional configuration we should discuss the distortion characteristics of a flexible tail plane. For this model we take a rigid wing (symmetrical section, zero angle relative to fuselage), rigid fuselage and a rigid tail plane with a flexible mounting of torsional stiffness M_θ. (lb ft/rad) between the tail plane and fuselage. This is shown in Figure 3.

In steady trimmed level flight

$$L_W = W\frac{l_T}{l_W + l_T} = \tfrac{1}{2}\rho V^2 S_W a_0 \alpha \tag{7}$$

$$L_T = W\frac{l_W}{l_W + l_T} = \tfrac{1}{2}\rho V^2 S_T \left(a_1[\alpha + \theta - \epsilon] + a_2\eta\right) \tag{8}$$

The structural equilibrium of the tail is given by

$$L_T ec - M_T = m_\theta \theta \tag{9}$$

where M_T is the moment about the tail aerodynamic centre (+ nose down) due to the elevator deflection η defined by

$$M_T = \tfrac{1}{2}\rho V^2 S_T c_T m \eta \tag{10}$$

The effect of M_T on the overall equilibrium conditions is usually neglected. The substitution of equations (9) and (10) into Equation (8) gives the elevator angle to trim.

$$\eta = \frac{[L_T - \tfrac{1}{2}\rho V^2 S_T a_1 \alpha (1 - \partial\epsilon/\partial\alpha)]}{\tfrac{1}{2}\rho V^2 S_T a_2} \; \frac{[M_\theta - \lambda \tfrac{1}{2}\rho V^2 S_T c_T a_1 m/a_2]}{[M_\theta - \tfrac{1}{2}\rho V^2 S_T c_T a_1 m/a_2]} \tag{11}$$

where

$$\lambda = \frac{ea_2}{m} \, \frac{L_T}{L_T - \tfrac{1}{2}\rho V^2 S_T a_1 \alpha (1 - \partial\epsilon/\partial\alpha)}$$

The elevator angle to trim tends to infinity as $V \rightarrow V_c$ where

$$V_c = \left[\frac{M_\theta}{\tfrac{1}{2}\rho S_T c_T a_1 m/a_2}\right]^{\frac{1}{2}} \tag{12}$$

The speed V_c is therefore the maximum speed theoretically possible for trimmed level flight. Alternatively, if the elevator effectiveness for trim is defined as the elevator required to trim (i.e. Equation 11) divided by the elevator required to trim with a completely rigid tail (i.e. Equation 11 with m_θ infinite), then V_c is the speed at which there is no control effectiveness to trim. This is sometimes referred to as the 'elevator reversal speed' but this definition is not really admissible since the aircraft cannot attain the speed at which it cannot be controlled, and so the concept of reversal of control, which implies flight through the critical region, is not relevant.

Note that when $\lambda = 1$ there is no loss of control effectiveness except at the critical speed V_c. Therefore from a *trim point of view* tail flexibility is the critical parameter which determines the overall aeroelastic limiting conditions for this controlled condition of flight.

5. MODEL 4. REPRESENTING AN INTEGRATED AIRCRAFT

We now consider an idealized integrated system in which the wing, fuselage and tail are combined together, as they would be for a slender configuration with the aerodynamic loading distributed over the whole of the length. We take as our model a simple beam of uniform cross-section (uniform bending stiffness distribution EI, uniform weight distribution σ lb/ft, total length l) under a simplified linear aerodynamic load distribution which is assumed to be $K\rho V^2 \alpha(x)$, where $\alpha(x)$ is the local incidence and K is taken to be constant. To simulate the force on the elevator control, a force P is applied at the trailing edge of the system, ensuring

that the overall equilibrium conditions (Lift = Weight, Moment = 0) are satisfied. This system is shown in Figure 4.

In steady trimmed flight the distortion function $y(x)$ satisfies the differential equation

$$EI\, y^{iv} = -\sigma - K\rho V^2 y' \tag{13}$$

The boundary conditions are

$$y'''\big|_{x=0} = y''\big|_{x=0} = y\big|_{x=l} = 0 \tag{14}$$

$$y'''\big|_{x=l} = -\frac{P}{EI} \tag{15}$$

It can be shown that any solution of Equation (13) which satisfies the boundary conditions (14) and (15) automatically satisfies the overall equilibrium conditions.

The solution of Equation (13) is

$$-y' = \alpha(x) = \frac{\sigma}{K\rho V^2} + Ae^{\lambda x} + e^{-\frac{1}{2}\lambda x}\left(B\cos\frac{\sqrt{3}}{2}\lambda x + C\sin\frac{\sqrt{3}}{2}\lambda x\right) \tag{16}$$

where

$$\lambda^3 = -\frac{K\rho V^2}{EI} \tag{17}$$

and A, B and C are arbitrary constants. The substitution of Equation (16) into the three boundary conditions (14) gives three simultaneous equations for A, B and C (the coefficients of which depend on λ). Thus the general solution is

$$A = B = C = 0$$

except at a critical velocity V_c, corresponding to the value of λ when the determinant of the simultaneous equations is zero. At this speed A, B and C are indeterminate, although B and C can be found in terms of A.

Equation (15) gives the control force P once the distortion is calculated. Thus in general P is zero except at $V = V_c$ where P depends on the coefficient A (assuming that B and C have been calculated in terms of A).

Therefore, for this model in the trimmed state there is no distortion until a critical speed is reached when a distortion can exist, depending on the size of an arbitrary control force P. The absence of distortion for speeds below V_c is explained physically in the present model by the fact that the lift distribution exactly balances the weight distribution. In order to deduce further information from the model it is necessary to alter this balance of aerodynamic and weight loadings. This may be done by modifying the uniform weight distribution in this model

(as shown in Fig.4) to an asymmetric weight distribution. If it is now assumed that the weight distribution is taken to be

$$\sigma = \sigma_0\left(2 - \frac{x}{\rho}\right) \tag{18}$$

then the distortion function $y(x)$ satisfies the differential equation

$$EI\,y^{iv} = -\sigma_0\left(2 - \frac{x}{\rho}\right) - K\rho V^2 y' \tag{19}$$

The boundary conditions are the same as before, i.e. Equations (14) and (15). The solution of Equation (19) is

$$-y' = \alpha(x) = \frac{\sigma_0\,(2 - x/l)}{K\rho V^2} + Ae^{\lambda x} + e^{-\frac{1}{2}\lambda x}\left(B\cos\frac{\sqrt{3}}{2}\lambda x + C\sin\frac{\sqrt{3}}{2}\lambda x\right) \tag{20}$$

where λ is the same as before (i.e. Equation 17). The substitution of Equation (20) into the boundary conditions (14) again gives three simultaneous equations for A, B and C which in this case gives finite non-zero values for these coefficients for each value of speed (or λ). The control force P can then be determined by substituting these known values of A, B and C into Equation (20) and then satisfying condition (15). However, A, B and C increase as λ approaches the critical value at which the determinant of coefficients of A, B and C in the simultaneous equation is zero. At the critical speed V_c the distortions tend to infinity as the control force P becomes very large.

It is noted that the critical speed is the same for both the weight distributions. Therefore the critical speed is independent of the weight distribution but the control effectiveness to trim below this critical maximum trim speed depends on the out-of-balance of the aerodynamic lifting and weight distributions.

It could be argued that the aeroelastic effects associated with the tail flexibility on conventional aircraft are similar to the overall effects of the whole of the integrated slender aircraft.

6. THE SLENDER CONFIGURATIONS

The previous simplified models have given a qualitative idea of the main aeroelastic effects which we might expect to occur. At this stage therefore it is necessary to extend these models, especially those representing the integrated configurations, to incorporate the particular features of the slender aircraft, both structurally and aerodynamically. The slender configuration is shown in Figure 5, together with the associated notation.

It is assumed that this aircraft will bend as a beam. For a given structure the bending stiffness distribution $EI(x)$ and the lengthwise weight distribution $w(x)$ are assumed to be known. The main difficulty at this stage is the general lack of knowledge of the aerodynamic loading on streamwise cambered slender wings. However,

it is assumed in this Report that the loading distribution $l(x)$ is given by the equation

$$l(x) \;=\; \rho V^2 \frac{d}{dx}\left[\pi s^2(x)\,\alpha(x) + 4\,\alpha(x)\,|\alpha(x)| \int_0^x s(x)\,dx\right] \tag{21}$$

for the general shape wing (i.e. $s(x)$ and incidence distribution $\alpha(x)$). It is assumed here that $\alpha = 0$ is the zero lift case. This is an intuitive approximation based on the following observations:

(i) If the non-linear terms are absent (i.e. if there are no separation and associated vortices) then the loading is given by linearized slender-wing theory which states that

$$l(x) \;=\; \rho V^2 \frac{d}{dx}\,[\pi s^2(x)\,\alpha(x)]$$

for general distributions $s(x)$ and $\alpha(x)$.

(ii) The only reliable formula incorporating the leading-edge vortices, based on both theoretical and experimental grounds, is for plate (or symmetrical) delta aerofoils. This formula is

$$l(x) \;=\; \rho V^2\,[2\pi k^2\,x\,\alpha + 4\,k\,x\,\alpha|\alpha|\,]$$

where $s(x) = kx$ and $\alpha(x) = \alpha$ which is constant.

Equation (21) is correct for these two particular cases and it is hoped that its generalized form will give a useful loading formula which represents the essential loading characteristics of deformed slender shapes.

The overall equilibrium conditions which must be satisfied in the trimmed state are

$$\int_0^c [l(x) - w(x)]\,dx + P \;=\; 0 \tag{22}$$

and

$$\int_0^c [l(x) - w(x)]\,[x - c]\,dx \;=\; 0 \tag{23}$$

The differential equation for the deformed aircraft shape $\dot{z}(x)$ is

$$\frac{d^2}{dx^2}\,(EI\,\dot{z}'') \;=\; -\rho V^2 \frac{d}{dx}\left[\pi s^2\,\dot{z}' + 4\dot{z}'\,|\dot{z}'|_1 \int_0^x s(x)\,dx\right] - w(x) \tag{24}$$

with the boundary conditions

$$EI\,\ddot{\vartheta}'''\big|_{x=0} = EI\,\ddot{\vartheta}''\big|_{x=0} = \frac{d}{dx}(EI\,\ddot{\vartheta}'')\big|_{x=0} = 0 \tag{25}$$

$$\frac{d}{dx}(EI\,\ddot{\vartheta}'')\big|_{x=0} = -P \tag{26}$$

This Equation (24) may be regarded as a third-order equation for the incidence distribution $\alpha(x)$ satisfying the three boundary equations (25). From this solution the control force P for overall equilibrium is then calculated from Equation (26). On integration, applying the appropriate boundary conditions Equation (24) becomes

$$EI\,\alpha'(\xi) = -\rho V^2 c^2 \int_0^{\xi} \left[\pi s^2(\xi)\,\alpha(\xi_1) + 4\alpha(\xi_1)\left|\alpha(\xi_1)\right| \int_0^{\xi_1} s(\bar{\xi})\,d\bar{\xi}\right] d\xi_1 \tag{27}$$

$$+ c^3 \int_0^{\xi}\int_0^{\xi_1} w(\bar{\xi})\,d\xi_1\,d\bar{\xi}$$

where $\xi = x/c$

with the boundary condition

$$EI\,\alpha'\big|_{\xi=1} = 0 \tag{28}$$

Equation (27) is completely general, and it can be solved for any distribution EI, $w(\xi)$, $s(\xi)$ etc. by a simple step by step procedure from $\xi = 0$ to $\xi = 1$.

However, to illustrate the main qualitative trends from this equation we shall concentrate once again on some simple slender models. These are taken to be slender flat plates, first a simple delta and the second a gothic outline. Although these plates do not have the type of distributions of weight and stiffness which are expected in a typical supersonic project they are, at least, consistent in their relationship between stiffness and weight and they will show the main aeroelastic trends of distortion in trimmed flight.

7. SLENDER DELTA PLATE WING

We now consider the simple case of a slender delta wing of constant thickness t and specific weight σ per net area (lb/ft^2) trimmed for level flight by a control force P at the trailing edge. In this case therefore

$$s(x) = Kx = Kc\xi \tag{29}$$

The substitution of the appropriate stiffness and weight distributions into the differential Equation (28) gives

$$\xi\alpha' = -\Lambda\int_0^{\xi}\xi^2\alpha\left(1+\frac{2|\alpha|}{\pi K}\right)d\xi_1 + \frac{\Lambda_1\xi^3}{3} \tag{30}$$

where

$$\Lambda = 6\pi(1-\nu^2)\left(\frac{\rho V^2}{E}\right)\left(\frac{c}{t}\right)^3 K$$

$$\Lambda_1 = 6\,(1-\nu^2)\left(\frac{\sigma}{E}\right)\left(\frac{c}{t}\right)^3$$

The boundary condition (28) is

$$\alpha'(1) = 0 \tag{31}$$

We may consider this problem in two parts, namely with and without the non-linear part of the aerodynamic loading, that is, considering the problem in the presence of, and in the absence of, the leading edge vortices.

(i) *Linear Aerodynamics*

The differential equation in this case is

$$\xi\alpha' = -\Lambda\int_0^{\xi}\xi_1^{\,2}\alpha\,d\xi_1 + \frac{\Lambda_1\xi^3}{3} \tag{32}$$

and $\alpha(\xi)$ must satisfy the end condition given by Equation (31). The first obvious solution which satisfies the boundary condition is that α is constant and is given by

$$\alpha = \frac{\Lambda_1}{\Lambda} \tag{33}$$

This solution, however, is not unique. This is shown by obtaining the general series solution of Equation (32), which is

$$\alpha = \alpha_0 - (\Lambda\alpha_0 - \Lambda_1)\left(\frac{\xi^3}{3^2} - \frac{\Lambda\xi^6}{3^2.6^2} + \frac{\Lambda^2\xi^9}{3^2.6^2.9^2}\right)$$

Boundary condition (31) gives

$$(\Lambda\alpha_0 - \Lambda_1)\left(\frac{1}{3} - \frac{\Lambda}{3^2.6} + \frac{\Lambda^2}{3^2.6^2.9}\right) = 0 \tag{34}$$

Therefore α is constant, given by Equation (33), except at the critical speed (or Λ) when the second bracket in Equation (34) is zero, and at this speed α_0 is

arbitrary. This implies that the control force P is zero except at the critical speed where any control force P may be applied, keeping the plate in trim.

This problem is therefore equivalent to Model 4, since physically the weight and aerodynamic lifting distributions again exactly balance. The critical speed from Equation (34) is given by

$$\Lambda = 33 \tag{35}$$

(ii) *Non-Linear Aerodynamics*

The differential equation in this case is the full Equation (30), together with the boundary condition (31). Again in this case a trivial solution exists giving α constant, where α is given by

$$\alpha\left(1 + \frac{2|\alpha|}{\pi K}\right) = \frac{\Lambda_1}{\Lambda} \tag{36}$$

Physically this is because the presence of the vortices, according to the present aerodynamic assumption (i.e. Equation 21), do not alter the *distribution* of loading; they only increase the intensity of loading. So the plate takes up a smaller angle of incidence, without distortion. And again solution (36) is not unique. A series expansion can be applied, assuming $\alpha > 0$ for $0 \leqslant \xi \leqslant 1$, giving a solution of the following form

$$\alpha = \alpha_0 - \left[\Lambda\alpha_0\left(1 + \frac{2|\alpha_0|}{\pi K}\right) - \Lambda_1\right] F(\xi, \Lambda, \Lambda_1, \alpha_0) \tag{37}$$

The boundary condition (31) gives the result that

$$\left[\Lambda\alpha_0\left(1 + \frac{2|\alpha_0|}{\pi K}\right) - \Lambda_1\right] F'(1, \Lambda, \Lambda_1, \alpha_0) = 0 \tag{38}$$

Therefore, at each speed, in addition to the solution (36), there is the possibility of another distortion shape if

$$F'(1, \Lambda, \Lambda_1, \alpha_0) = 0 \tag{39}$$

There seems to be at least one simple solution of Equation (39) so that in general at each speed there appear to be two distinct positions of equilibrium. As the speed increases these two distortion shapes merge together until a certain speed is reached at which both distortion shapes coincide. Above this speed the two distortion shapes diverge from each other. This speed at which the two shapes coincide might be regarded as a critical speed, near which the aircraft would tend to jump from one position of equilibrium to the other, assuming that *both positions of equilibrium are stable*. It could be argued that, in the region of this critical speed, if one position of equilibrium is stable and the two positions gradually coincide with increase of speed the other position of equilibrium is stable. This question of stability cannot be resolved at this stage and requires further investigation before a definite conclusion may be reached about the relationship of these positions of

equilibrium. In this Report we have only established their existence. The critical velocity of this plate, from Equation (38) comes out to be

$$\Lambda = 19.1 \tag{40}$$

It is interesting to note the low value of this speed compared to the critical speed in the absence of the leading-edge vortices (Equation 35).

The equilibrium shapes at $\Lambda = 16.5$ and $\Lambda = 22$ are shown in Figure 6.

8. SLENDER GOTHIC PLATE WING

This gothic plate wing is shown in Figure 7. Again this problem was investigated with and without the non-linear aerodynamic terms; in this problem the step by step numerical techniques, mentioned earlier, had to be applied. The results are summarized below.

(i) *Linear Aerodynamics*

The maximum trim speed in this case is given by

$$\Lambda = 44.5 \tag{41}$$

The variation of the equilibrium shapes of the wing at values of below the critical, together with the variation of control force, are shown in Figure 8.

(ii) *Non-Linear Aerodynamics*

It was found in this case that at low values of Λ two positions of equilibrium are apparently possible. This is a similar result to the delta plate. At a critical speed, given by

$$\Lambda = 21.6 \tag{42}$$

both positions of equilibrium coincide to a single position. Above this critical speed *no positions of equilibrium are* apparently possible. Shapes of equilibrium are shown in Figure 9.

9. CONCLUSIONS

The main conclusions regarding the aeroelastic effects in trimmed level flight may be summarized as follows:

(i) On a conventional aircraft it is primarily the tail flexibility which is responsible for the loss of control effectiveness to trim resulting in a maximum trim speed.

(ii) The assumption of linear aerodynamics on an integrated aircraft results in a loss of control effectiveness to trim with increasing speed, giving a

maximum trim speed (i.e. zero control effectiveness) at which the distortion of the whole airframe is very large. This limiting or critical speed is independent of the weight distribution, but the control effectiveness effectiveness to trim depends on the out-of-balance of the aerodynamic and weight distributions.

(iii) From the assumption of non-linear aerodynamics on an integrated aircraft it appears that at low speeds two independent positions of equilibrium exist. These two modes of distortion coincide at a certain critical speed. Above this critical speed there may be either two positions of equilibrium or no positions of equilibrium. The nature of this critical speed has not been satisfactorily explained in this Report; this can only be done by reference to the stability of the various modes of equilibrium. This investigation is at present under consideration.

It is interesting to note that the critical speed with non-linear aerodynamics is considerably less than the critical speed with linear aerodynamics. This may be relevant to the current ideas on the performance of slender aircraft when the cruise condition is designed to have little or no flow separation at the leading edges so that the cruise is determined by linear aerodynamic considerations, whereas the off-cruise condition will have to contend with the effect of leading-edge vortices, that is the effects of non-linear aerodynamics.

Finally, this paper is an introductory attempt to understand some of the fundamental principles of aeroelasticity relating to the integrated aircraft. The approach advocated in this paper, of studying simple models, produces the general background which is desirable before any detailed design work is entered into.

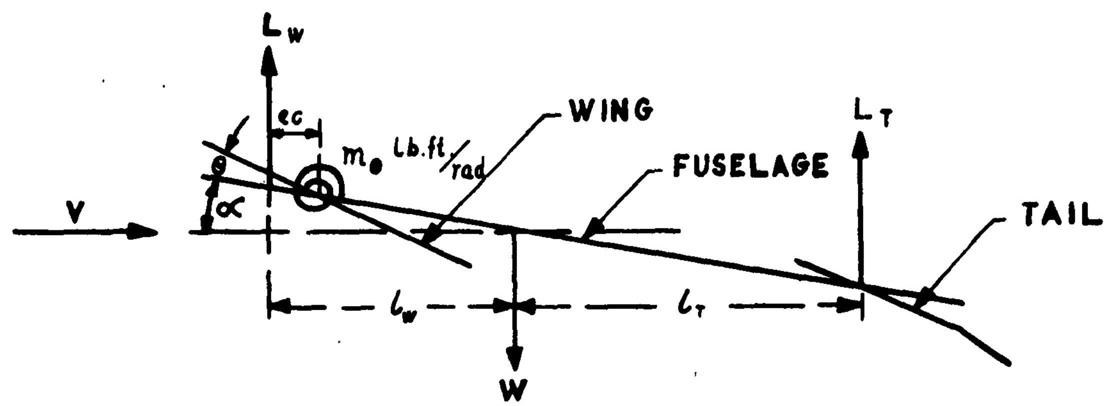

Fig. 1 Classical aircraft - flexible wing

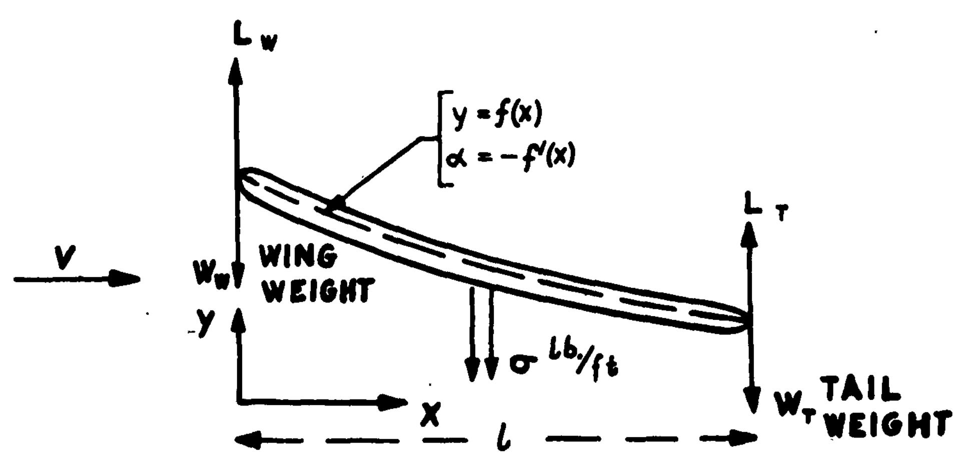

Fig. 2 Classical aircraft - flexible fuselage

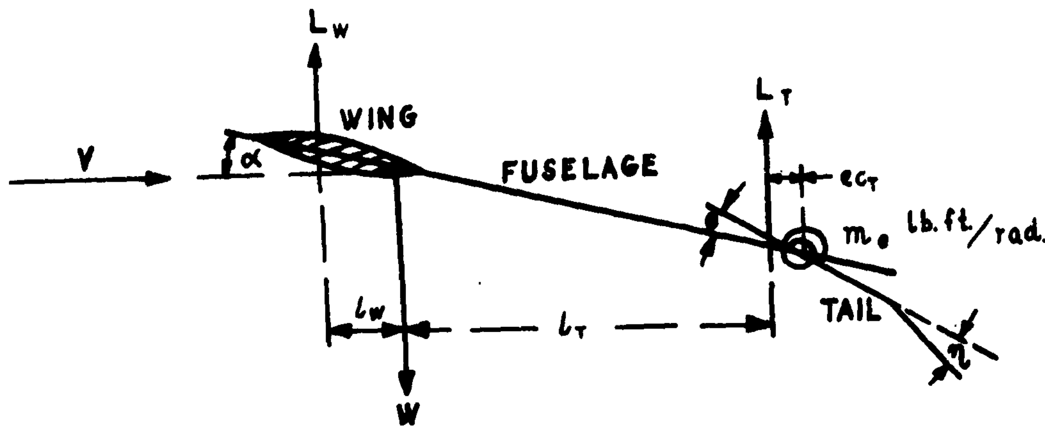

Fig.3 Classical aircraft - flexible tail

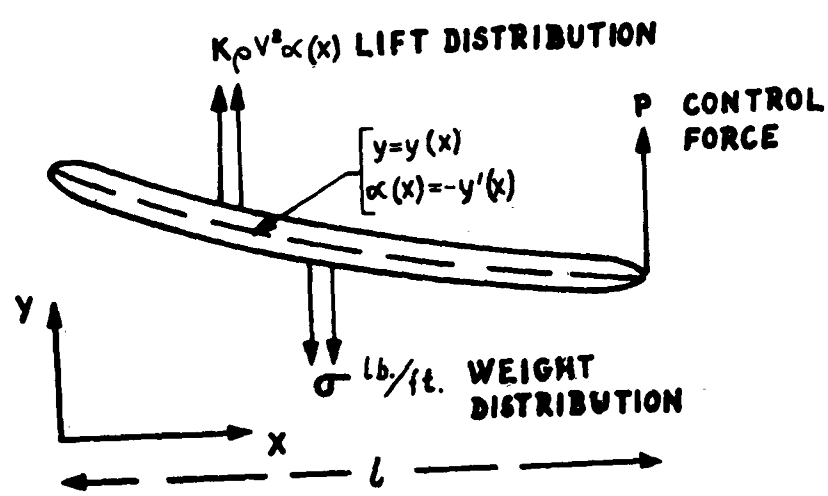

Fig.4 Simple integrated aircraft

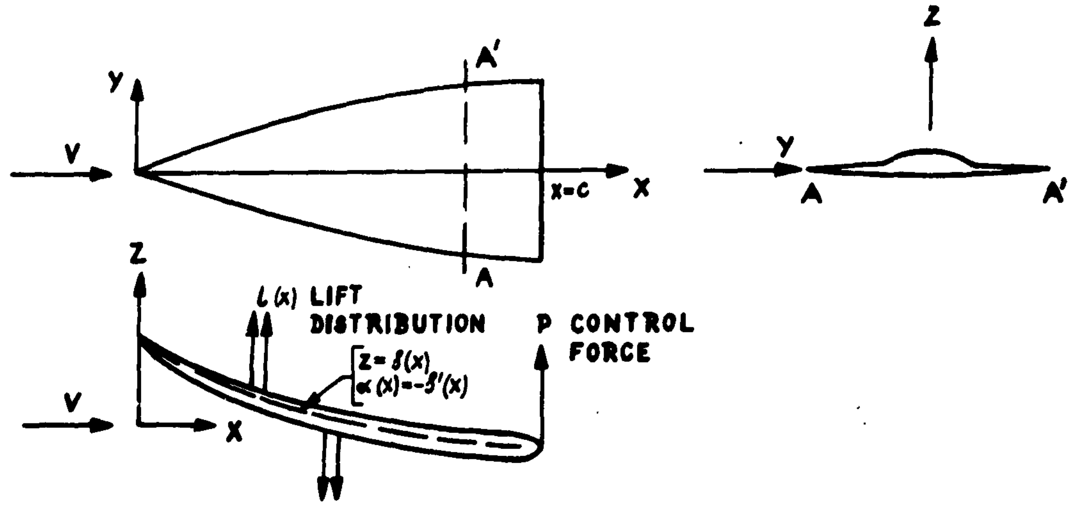

Fig.5 Notation for slender aircraft

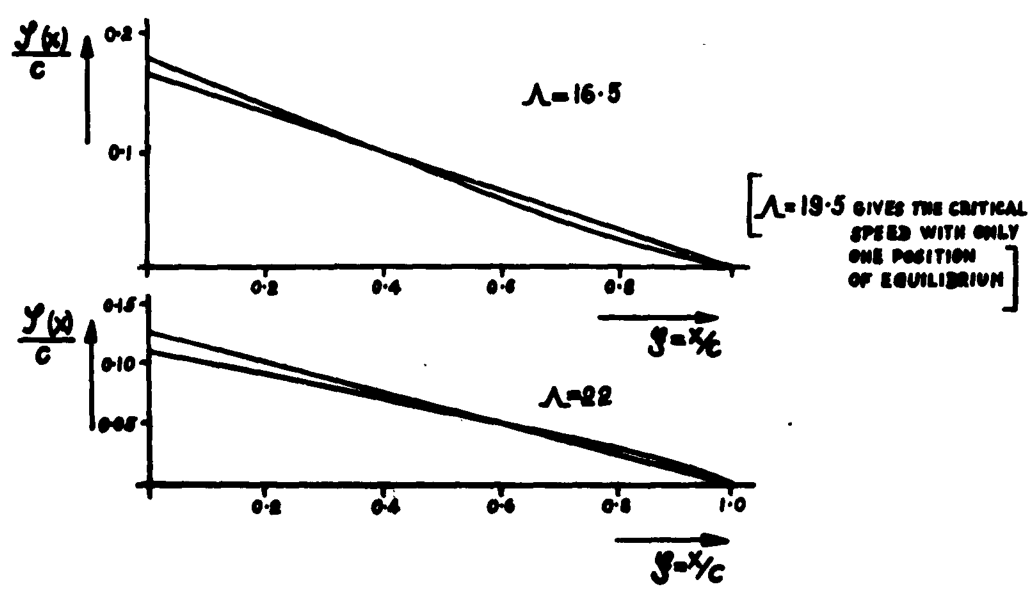

Fig.6 Equilibrium positions of delta plate - non-linear aerodynamics

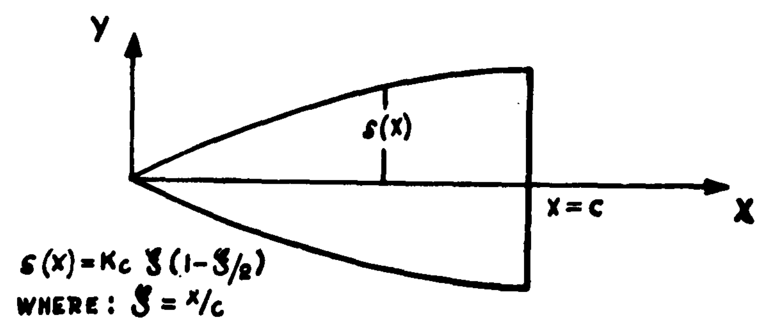

Fig.7 Gothic plate wing

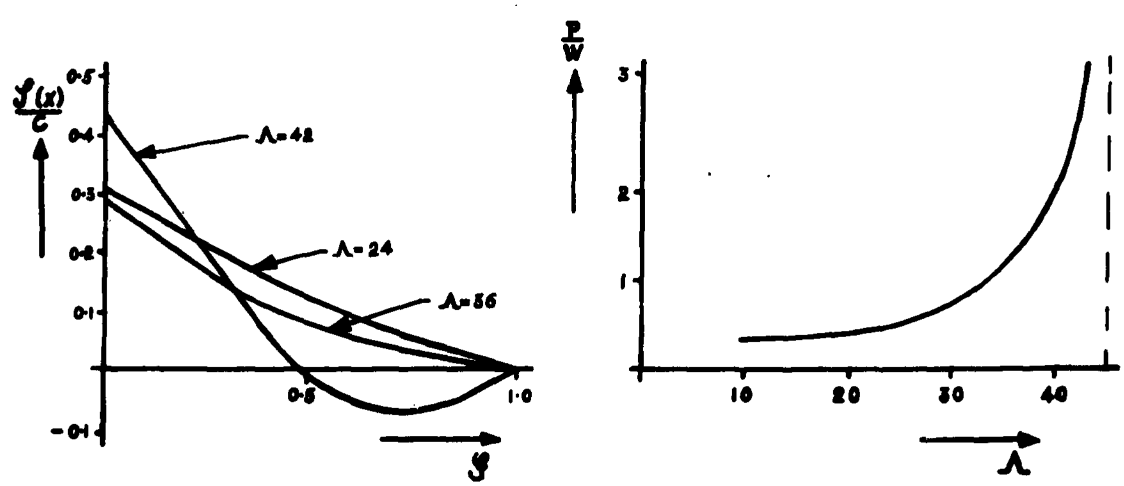

Fig.8 Equilibrium shapes of gothic plate - linear aerodynamics

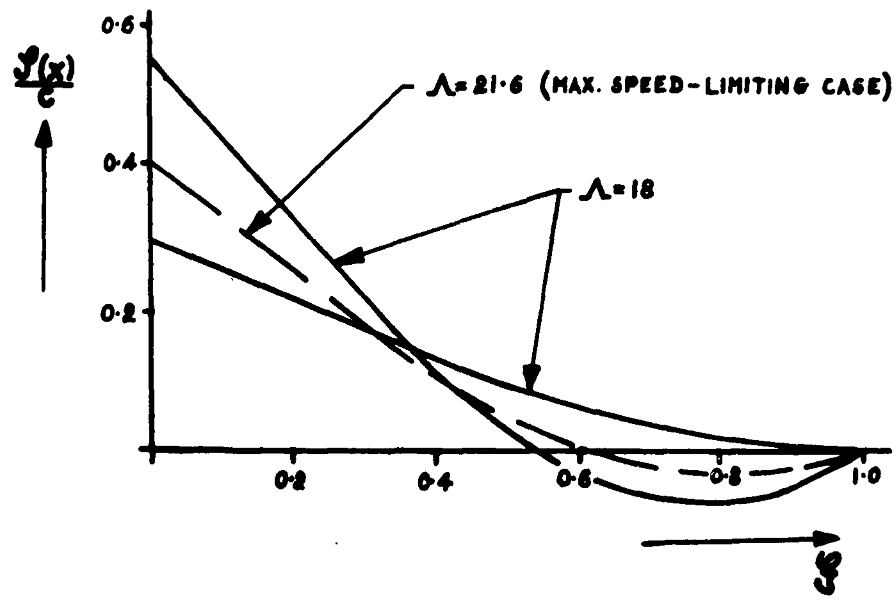

Fig.9 Equilibrium shapes of gothic plate - non-linear aerodynamics

DISCUSSION

G.H. Lee (U.K.): Mr. Hancock has shown that with linear aerodynamics a flat plate of gothic planform can have, in general, two different equilibrium shapes, though at a certain speed these two shapes are the same. However, with non-linear aerodynamics no solution is possible at speeds beyond that for which the two shapes coincide; he therefore calls this a critical speed.

Since the difference in these two cases is due to the incorporation of non-linear terms, I should like to know if Mr. Hancock has any idea as to how sensitive this speed is to the type of non-linear terms incorporated. In other words, would you get similar critical speeds with different but reasonable non-linear theories, or is the critical speed going to be very sensitive to the type of theory used?

Reply by Author: First, I must correct the misapprehension gained by Mr. Lee from my lecture; with linear aerodynamics there is only one position of equilibrium at each speed, it is only with non-linear aerodynamics that there arises the possibility of more than one position of equilibrium. However, my paper is intended to be only a qualitative introduction to the type of aeroelastic phenomena which may arise on slender aircraft in the presence of non-linear aerodynamic forces; such critical speeds will occur with other assumptions for the non-linear aerodynamic loading although it is premature at this stage to give reliable quantitative assessments of these critical speeds. It should be remembered that quantitative results depend not only on a more reliable knowledge of the non-linear aerodynamics but also on more realistic estimates of the structural and inertial loadings.

H.H.B.M. Thomas (U.K.): Regarding Mr. Lee's question to Dr. Hancock, the point to note is that linear aerodynamics leads to the cruise as the critical condition aeroelastically with the flight path plan proposed for such aircraft. The query raised by the present paper is whether off-design condition at larger incidence may not be more critical because of non-linear aerodynamics. I am sure Dr. Hancock will agree that his paper points to the existence of a possible problem rather than an assessment of the severity of the problem in an actual design.

ADDENDUM

AGARD SPECIALISTS' MEETING

on

STABILITY AND CONTROL

Complete List of Papers Presented

Following is a list of the titles and authors of the 41 papers presented at the Stability and Control Meeting held in Brussels in April, 1960, together with the AGARD Report number covering the publication of each paper.

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1961
17 pages incl. 9 figs; plus discussion and bibliography of papers presented at the Stability and Control Meeting

A series of simple theoretical models are discussed, representing the essential features of flexible aircraft both of the classical and integrated varieties, in trimmed level flight. Assuming linear aerodynamics it is shown that the maximum

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speed for trimmed flight for the classical aircraft is determined primarily by the tail-plane flexibility. For the integrated aircraft this maximum speed occurs when both the overall aeroelastic distortion and the control forces become very large, the effectiveness of the control to trim is related to the similarity of the weight and aerodynamic lifting distributions. Detailed calculations on trimmed plate wings of varying plan form further illustrate these general points.

The aeroelastic effects of these trimmed plate wings in level flight are also investigated assuming non-linear aerodynamics. It is shown that in general there are two positions of equilibrium of the aircraft at each speed although it is possible that at high speeds both these positions of equilibrium are imaginary. The stability of these positions of equilibrium are not discussed in this paper.

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